

# Extended model for monopole catalysis of nucleon decay

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A new model for monopole catalysis of nucleon decay is proposed. Unlike in the earlier one, the only light fields in this model are the photon and Skyrme (pion) field. The model admits the 't Hooft–Polyakov monopole and Skyrmion as classical solutions, while baryon number nonconservation occurs through an anomaly involving an intermediate mass axial vector field resembling  $W$  and  $Z$  bosons. By considering spherically symmetric monopole-Skyrmion configurations, we find that the Skyrmion loses essentially all its mass when interacting with the monopole, which is phenomenologically identical to the monopole catalysis of nucleon decay. Short-distance monopole-Skyrmion physics in this model is interesting too, as there exist almost degenerate metastable monopole-Skyrmion bound states separated by substantial energy barriers. Yet the height of the latter is much smaller than the physical nucleon mass, so the complete disappearance of the normal undeformed Skyrmion remains perfectly possible in Skyrmion-monopole scattering.

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## I. INTRODUCTION

Models admitting both a 't Hooft–Polyakov monopole [1,2] and Skyrmion [3] as static solutions to field equations are of interest from various points of view. One aspect is the possibility to describe the monopole catalysis of nucleon decay [4,5] at purely a classical level [6,7]. In grand unified theories, the dynamics of the monopole catalysis involves several widely separated length scales, the smallest of which is the size of the monopole core, roughly of the order of  $1/v$ , where  $v$  is the grand unified theory (GUT) vacuum expectation value (VEV),  $v \sim 10^{16}$  GeV, while the largest one is the hadronic scale of the order of  $1/F_\pi$ , where  $F_\pi \sim 100$  MeV. One of the sources of baryon number nonconservation, which may be relevant to the monopole catalysis, is the electroweak anomaly [8], with the corresponding length scale roughly of the order of  $1/\zeta$  where  $\zeta \sim 100$  GeV is the standard model VEV. The existence of so different scales, as well as the nonperturbative nature of dynamics, make any analysis of the monopole catalysis difficult.

A simple model aimed to describe the catalyzed decay of a nucleon, represented by the Skyrmion, in interactions with the 't Hooft–Polyakov monopole was suggested in Ref. [7]. That model, however, featured in its spectrum an axial vector boson with mass scale set by  $F_\pi$  rather than by the electroweak scale. This unrealistic property is particularly unpleasant in view of the fact that baryon number nonconservation in that model occurs via anomalous interactions of the Skyrme field to the axial vector boson, so one might worry that the Skyrmion decay in the presence of the monopole is unsuppressed precisely because the axial vector field is so light.

In the present work the simplified model of nucleon decay [7] is refined by addition of a new axial symmetry-breaking Higgs field in the same group representation as the Skyrme field itself. The VEV  $\zeta$  of the extra field introduces a new energy scale into the model which naturally resembles the

electroweak scale, and hence the new massive axial bosons resemble the  $W$  and  $Z$  bosons. Thus, the only light fields in the extended model are the Skyrme (pion) field and the electromagnetic field. In this sense the model is completely realistic.

In this paper we study static, spherically symmetric field configurations in the extended model. Our main findings are twofold. First, in complete analogy to Ref. [7] we observe that on top of the monopole, there are no static solutions with baryon number of order 1, whose mass and/or size are comparable to the mass and/or size of a nucleon (“bare” Skyrmion). This is directly relevant to the monopole catalysis: whatever happens to nucleons or quarks near the monopole core, the nucleon gets squeezed into a small region near the monopole, and its mass is irradiated away in the form of pions. This is phenomenologically identical to the catalyzed nucleon decay.

The second set of observations concerns the properties of our system at short distances, i.e., near the monopole core. Of course, the description of the strong interaction physics in terms of the Skyrme model is not valid at short distances, yet our results may be of interest for understanding the Skyrmion-monopole system per se. We find, in sharp contrast to Ref. [7], that the unimpeded decay of the static Skyrmion-like state is no longer possible; instead we find a sequence of Skyrmion-monopole bound states separated by energy barriers, the picture similar to the familiar periodic structure of topological vacua in gauge theories. The energy minima correspond to integer baryon charge and their energies (referenced from the mass of the “bare” monopole) are of order  $F_\pi^2/(gv)$ , up to logarithm, where  $g$  is the gauge coupling constant. The height of the energy barrier at half-integer baryon charge is defined by the solution involving the extra Higgs field, analogous to the sphaleron solution [9]. At  $\zeta \gg F_\pi$ , where  $\zeta$  is the intermediate energy scale (VEV of the extra Higgs field), the barrier height is of order  $\zeta^2/(gv)$ . Even though the barrier height is large compared to the

masses of metastable bound states, all energies involved are tiny compared to the physical nucleon mass, provided the realistic hierarchy

$$F_\pi \ll \zeta \ll v$$

holds. This indeed demonstrates that the nucleon loses almost all its energy when interacting with the monopole.

For the sake of completeness we consider also the unphysical case of large  $\zeta/v$ . In that case the extra Higgs field modifies the Chern-Simons number in a nontrivial way, resulting in the multiple valued dependence of the baryon number  $B$  on the asymptotic value of the Skyrme field  $f_\infty$ . The dependence of energy  $E$  on  $B$  is still uniquely defined, with sharp minima at integer  $B$ , but it is no longer possible to identify a single well-defined barrier separating these minima.

It is worth noting that the static results obtained both in the simplified model [7] and its extended version described below are not directly applicable to the kinetics of real Skyrmin decay, once the latter is not even a solution in the static models. The object stabilized by the introduction of the intermediate scale  $\zeta$  is a static Skyrmin-like solution localized on the monopole core with energy negligible compared to that of real Skyrmin. In other words, the Skyrmin decay is a nonstationary process; we hope to study its dynamics in a future publication.

## II. THE MODEL

### A. Field content

As explained in Ref. [7], our model is designed to leave an unbroken (electromagnetic)  $U(1)$ . It consists of the Skyrme field  $U$  coupled to two gauge fields

$$SU(2)_L: A_\mu,$$

$$SU(2)_R: B_\mu.$$

In addition to the two Georgi-Glashow Higgs fields  $\Phi_A$  and  $\Phi_B$  which break the  $SU(2)_L$  and  $SU(2)_R$  symmetries down to  $U(1)_L$  and  $U(1)_R$ , respectively, at the “GUT” scale  $v$ , we add a third Higgs field  $\Psi$  which breaks the axial subgroup of the remaining  $U(1)_L \times U(1)_R$ . In the original model [7] the axial  $U(1)$  had been broken by the Skyrme field itself, with the corresponding vector boson mass of the order of  $gF_\pi$ . Here the scale of axial symmetry breaking is fixed by the vacuum expectation value of the  $\Psi$  field  $\zeta$ , which we assume to be an intermediate scale between  $v$  and  $F_\pi$ . To break the axial  $U(1)$  only, the  $\Psi$  field must be in the same group representation as the Skyrme field. The latter requirement fixes the scalar field content of the updated model completely:

$$\begin{aligned} U &\in SU(2), \quad U, \Psi: (2, \bar{2}), \\ \Phi_A &: (3, 1), \\ \Phi_B &: (1, 3). \end{aligned} \quad (1)$$

As in Ref. [7], we take the gauge couplings, triplet Higgs self-couplings and their vacuum expectation values to be the same in the left ( $L$ ) and right ( $R$ ) sectors

$$g_A = g_B \equiv g, \quad \lambda_A = \lambda_B \equiv \lambda, \quad v_A = v_B \equiv v.$$

The action for this model is

$$\begin{aligned} S = \int d^4x & \left[ -\frac{1}{2g^2} \text{Tr}(F_{\mu\nu}^2) - \frac{1}{2g^2} \text{Tr}(G_{\mu\nu}^2) \right] \\ & - \int d^4x \left[ \frac{1}{2} \text{Tr}(D_\mu \Phi_A)^2 + \frac{1}{2} \text{Tr}(D_\mu \Phi_B)^2 \right] \\ & - \int d^4x \left[ \frac{\lambda}{4} \left( \frac{1}{2} \text{Tr}(\Phi_A^2) + v^2 \right)^2 + \frac{\lambda}{4} \left( \frac{1}{2} \text{Tr}(\Phi_B^2) + v^2 \right)^2 \right] \\ & + \int d^4x \left[ \frac{1}{2} \text{Tr}(D_\mu \Psi^\dagger D_\mu \Psi) + \frac{\tau}{4} \left( \frac{1}{2} \text{Tr}(\Psi^\dagger \Psi) - \zeta^2 \right)^2 \right] \\ & + \int d^4x \left[ -\frac{F_\pi^2}{16} \text{Tr}(U^\dagger D_\mu U)^2 \right. \\ & \left. + \frac{1}{32e^2} \text{Tr}([U^\dagger D_\mu U, U^\dagger D_\nu U]^2) \right] + \Gamma_{\text{WZW}}. \end{aligned} \quad (2)$$

Here  $F_{\mu\nu}$  and  $G_{\mu\nu}$  are the field strengths of  $A_\mu$  and  $B_\mu$ , the fields  $\Phi_A$  and  $\Phi_B$  are  $2 \times 2$  matrices from the algebras of  $SU(2)_L$  and  $SU(2)_R$ , respectively,

$$D_\mu U = \partial_\mu U + A_\mu U - U B_\mu,$$

$$D_\mu \Phi_A = \partial_\mu \Phi_A + [A_\mu, \Phi_A],$$

$$D_\mu \Phi_B = \partial_\mu \Phi_B + [B_\mu, \Phi_B], \quad (3)$$

and  $F_\pi$  and  $e$  are the pion decay constant and the Skyrme constant. Generally speaking, the new Higgs field  $\Psi$  is an arbitrary  $2 \times 2$  complex matrix with covariant derivative identical to that of the Skyrme field:

$$D_\mu \Psi = \partial_\mu \Psi + A_\mu \Psi - \Psi B_\mu.$$

As in Ref. [7], in what follows we can completely ignore the Wess-Zumino-Witten term  $\Gamma_{\text{WZW}}$  in Eq. (2).

### B. Masses and energies

For the physical hierarchy  $v \gg \zeta \gg F_\pi$ , the physical spectrum of the system decouples into a set of well-defined objects with masses of relevant scales. The heaviest mass scale is controlled by  $v$ —the vacuum expectation value of the “GUT” Higgs fields  $\Phi_A$  and  $\Phi_B$ . Here we have “GUT” Higgs and vector boson masses,

$$m_V = 2gv,$$

$$m_H = \sqrt{\lambda}v,$$

and the monopole mass,

$$M_{\text{mon}} = 4\pi D_{\text{mon}} \frac{v}{g}, \quad (4)$$

where  $D_{\text{mon}}$  at  $\lambda/g^2=0.5$  is around 2.4.

The next mass scale is related to  $\zeta$ —the vacuum expectation value of the extra Higgs field  $\Psi$ . In the proper vacuum state (see below) the extra field adds mass to the axial  $U(1)$  gauge boson only:

$$m_A = \left( g^2 \zeta^2 + \frac{g^2 F_\pi^2}{8} \right)^{1/2} \quad (5)$$

while the mass of the extra Higgs boson is

$$m_\Psi = \sqrt{\tau} \zeta. \quad (6)$$

The mass of the corresponding nonperturbative object would have been of the order of

$$M_{\text{sph}} \sim \frac{\zeta}{g}.$$

Once the axial gauge field is meant to mimic  $W$  and  $Z$  bosons, the nonperturbative object should be analogous to the sphaleron. As we will show below, this object indeed exists and defines the barrier height between local minima of static energy with integer baryon number. However, as will be shown below, in the presence of the monopole the corresponding unstable solution becomes much lighter than  $M_{\text{sph}}$ :

$$E_{\text{sph}} \sim \frac{M_{\text{sph}}^2}{M_{\text{mon}}} \sim \frac{\zeta^2}{gv}. \quad (7)$$

This is similar to what happens to the Skymion in our model (see also Ref. [7]). In the absence of pion mass potential term, the normal (“bare”) Skymion [3] would have mass on the  $F_\pi$  scale

$$M_{\text{Sk}} = \frac{4\pi}{e} D_{\text{Sk}} F_\pi, \quad D_{\text{Sk}} \sim 2.9$$

and radius

$$R_{\text{Sk}} \sim \frac{1}{e F_\pi}.$$

We will see that in the monopole background, the normal Skymion is no longer stable, which by itself points to the possibility of nucleon decay in the model at hand. The new “thin” Skymion (the static solution with unit baryon charge) is considerably smaller and lighter:

$$R'_{\text{Sk}} \sim \frac{1}{gv}, \quad (8)$$

$$E_{\text{Sk}} \sim \frac{M_{\text{Sk}}^2}{M_{\text{mon}}} \sim \frac{F_\pi^2}{gv}. \quad (9)$$

### C. Spherically symmetric ansatz

As in Ref. [7], we can simplify the ansatz because the action (2) is invariant under the spatial reflection  $x^0 \rightarrow x^0, \mathbf{x} \rightarrow -\mathbf{x}$  supplemented by the interchange of the left and right  $SU(2)$ , i.e.,

$$A_0 \leftrightarrow B_0, \quad A_i \rightarrow -B_i, \quad B_i \rightarrow -A_i,$$

$$\Phi_A \rightarrow -\Phi_B, \quad \Phi_B \rightarrow -\Phi_A, \quad U \rightarrow U^\dagger, \quad \Psi \rightarrow \Psi^\dagger \quad (10)$$

(all numerical solutions found are consistent with this symmetry). For static fields with  $A_0 = B_0 = 0$ , the most general spherically symmetric ansatz consistent with Eq. (10) is

$$A_i = -\frac{i}{2} \left[ \left( \frac{a_1(r)-1}{r} \right) \varepsilon_{ijk} \sigma_j \hat{x}_k + \left( \frac{a_2(r)}{r} \right) (\sigma_i - \hat{x}_i \hat{x} \cdot \vec{\sigma}) + \left( \frac{a_3(r)}{r} \right) \hat{x}_i \hat{x} \cdot \vec{\sigma} \right],$$

$$B_i = -\frac{i}{2} \left[ \left( \frac{a_1(r)-1}{r} \right) \varepsilon_{ijk} \sigma_j \hat{x}_k - \left( \frac{a_2(r)}{r} \right) (\sigma_i - \hat{x}_i \hat{x} \cdot \vec{\sigma}) - \left( \frac{a_3(r)}{r} \right) \hat{x}_i \hat{x} \cdot \vec{\sigma} \right],$$

$$\Phi_A = \Phi_B = i v h(r) \hat{x} \cdot \vec{\sigma},$$

$$U = \mathbb{1} \cos f(r) + i \hat{x} \cdot \vec{\sigma} \sin f(r),$$

$$\Psi = \zeta [i h_1(r) \hat{x} \cdot \vec{\sigma} + h_2(r) \mathbb{1}], \quad (11)$$

where  $\hat{x}$  is the unit radius vector.

It is worth noting that in the spherically symmetric ansatz (11) the field  $\Psi$  automatically breaks the axial subgroup of  $U(1)_L \times U(1)_R$  and preserves vectorial subgroup, so the photon remains massless. In less symmetric cases this happens only for a subset of all possible  $\Psi$ -field vacua, so it would be necessary to include additional terms into action to single out the correct symmetry breaking pattern yielding a massless photon.

With the ansatz (11), regularity of the Skyrme field at the origin requires that  $f(0)$  is an integer multiple of  $\pi$ ; without loss of generality we set

$$f(0) = \pi.$$

Other conditions, ensuring that the fields are regular at the origin, are

$$a_1(0) = 1, \quad a_2(0) = a_3(0) = 0, \quad h(0) = h_1(0) = h_2'(0) = 0.$$

Using the dimensionless coordinate

$$\rho = g v r$$

and neglecting the Wess-Zumino-Witten (WZW) term in Eq. (2), the static energy functional can be written as follows:

$$\begin{aligned}
H &= \frac{4\pi v}{g} \int d\rho \mathcal{E}(\rho), \\
\mathcal{E} &= \left[ \left( a_1' + \frac{a_2 a_3}{\rho} \right)^2 + \left( a_2' - \frac{a_1 a_3}{\rho} \right)^2 + \frac{1}{2\rho^2} (a_1^2 + a_2^2 - 1)^2 \right] \\
&+ 2 \left[ \rho^2 (h')^2 + 2(a_1^2 + a_2^2)h^2 + \frac{\lambda}{4g^2} \rho^2 (h^2 - 1)^2 \right] \\
&+ \left( \frac{\xi}{v} \right)^2 \left[ \rho^2 \left( h_1' - \frac{a_3}{\rho} h_2 \right)^2 + \rho^2 \left( h_2' + \frac{a_3}{\rho} h_1 \right)^2 + 2(a_1 h_1 \right. \\
&\left. - a_2 h_2)^2 + \frac{\tau}{4g^2} \left( \frac{\xi}{v} \right)^2 \rho^2 (h_1^2 + h_2^2 - 1)^2 \right] + \kappa_1 (X^2 + 2Y^2) \\
&+ \frac{4\kappa_2}{\rho^2} Y^2 (2X^2 + Y^2), \tag{12}
\end{aligned}$$

where the prime denotes the derivative with respect to  $\rho$ ,

$$\begin{aligned}
X &= \rho f' - a_3, \\
Y &= a_1 \sin f - a_2 \cos f, \tag{13}
\end{aligned}$$

and

$$\begin{aligned}
\kappa_1 &= \frac{F_\pi^2}{8v^2}, \\
\kappa_2 &= \frac{g^2}{64e^2}. \tag{14}
\end{aligned}$$

The radial energy functional (12) retains a residual gauge invariance, which is a remnant of the axial  $U(1)$ :

$$\begin{aligned}
h_2 + ih_1 &\rightarrow (h_2 + ih_1) e^{i\alpha(\rho)}, \\
a_1 + ia_2 &\rightarrow (a_1 + ia_2) e^{i\alpha(\rho)}, \\
a_3 &\rightarrow a_3 + \rho \partial_\rho \alpha(\rho), \\
f &\rightarrow f + \alpha(\rho). \tag{15}
\end{aligned}$$

Note that  $h_2 + ih_1$  rotates identically to  $e^{if}$ , once the fields  $\Psi$  and  $U$  are in the same group representation.

Making use of Eqs. (12) and (13), one finds the boundary conditions at  $\rho \rightarrow \infty$  from the requirement of finite energy. In the absence of the monopole we would have  $a_1 = 1, a_2 = 0$ , and hence  $f_\infty = \pi N$ . The latter condition would prevent the Skyrmeion from decaying, thus making the baryon number  $B$  topologically stable.

The presence of the monopole requires that the ‘‘GUT’’ Higgs fields be topologically nontrivial at infinity:

$$h(\infty) = 1$$

which immediately leads to  $a_1(\infty) = a_2(\infty) = 0$ , so that now the Skyrme field at infinity  $f_\infty$  can take any value, allowing for the Skyrmeion decay, while  $a_3(\infty) = (\rho f')_{\rho \rightarrow \infty}$ .

The boundary conditions for  $\Psi(\infty)$  require more careful consideration. The only condition following from Eq. (12) is

$$(h_1^2 + h_2^2)_{\rho \rightarrow \infty} = 1$$

or

$$(h_2 + ih_1)_{\rho \rightarrow \infty} = e^{i\beta}, \tag{16}$$

where  $\beta = 0$  in the vacuum. One can see immediately from Eq. (15) that the difference  $f_\infty - \beta$  is gauge invariant. Let us show that this quantity is also time independent in the dynamical version of the model. This is clear (see also Ref. [7]) in the  $a_0 = b_0 = 0$  gauge, where the canonical momenta are proportional to time derivatives of the fields. The kinetic energy of the Higgs and Skyrme fields contains the terms

$$\int r^2 dr \dot{h}^2, \quad \int r^2 dr (\dot{h}_1^2 + \dot{h}_2^2), \quad \text{and} \quad \int r^2 dr \dot{f}^2.$$

Therefore during any dynamical evolution the time derivatives of fields  $h, h_1, h_2$ , and  $f$  must vanish at infinity, so that their asymptotic values are time independent.<sup>1</sup> Once  $f_\infty - \beta$  is gauge invariant, it is time independent in any gauge, including the  $a_3 = 0$  gauge extensively used below.

Introduction of the new field  $\Psi$  does not alter the expression for baryon number  $B$ . In the spherically symmetric case one has

$$\begin{aligned}
B &= \int_0^\infty d\rho b(\rho), \\
b(\rho) &= \frac{1}{\pi} \frac{d}{d\rho} \left[ -f + \frac{1}{2} (a_1^2 - a_2^2) \sin 2f - a_1 a_2 \cos 2f \right] \\
&+ \frac{1}{\pi} \left[ (a_1 a_2' - a_1' a_2) - \frac{a_3}{\rho} (a_1^2 + a_2^2 - 1) \right], \tag{17}
\end{aligned}$$

which in the gauge  $a_3 = 0$  takes the form

$$B = \frac{\pi - f_\infty}{\pi} + \frac{1}{\pi} \int_0^\infty d\rho (a_1 a_2' - a_1' a_2). \tag{18}$$

Therefore, in the  $a_3 = 0$  gauge the baryon number  $B$  is parametrized by the free parameter  $f_\infty$ , with  $f_\infty = \pi$  at  $B = 0$ . By finding a set of static solutions corresponding to different values of  $f_\infty$ , we build a possible trajectory for quasistationary Skyrmeion decay (for more detailed discussion see Ref. [7]). Recalling that  $\beta - f_\infty$  is constant during the decay, we finally obtain the boundary condition for  $\Psi$ :

$$\beta = f_\infty - \pi. \tag{19}$$

<sup>1</sup>This is not the case for the field  $a_3$  which enters the kinetic energy through  $F_{0i} \propto \dot{a}_3 / r$ .

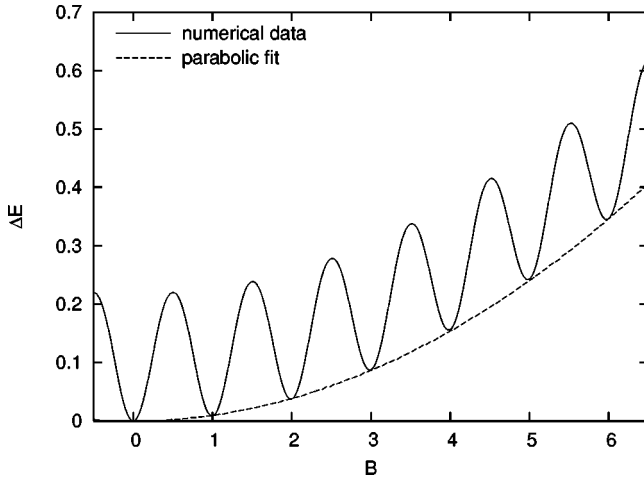


FIG. 1. Static energy in units of normal “bare” Skymion mass  $M_{\text{Sk}}$  as a function of the baryon number. Here  $F_\pi^2/8v^2 = 0.0001$ ,  $\zeta^2/v^2 = 0.2$ ; the energy is referenced from the monopole mass  $\Delta E = (E - M_{\text{mon}})/M_{\text{Sk}}$ .

As will be seen below, the condition (19) plays a crucial role. Without it, the  $\Psi$  field at any  $B$  remains close to its vacuum configuration and effectively decouples from the rest of the system whose properties would remain similar to those described in Ref. [7].

Finally, let us write the energy functional in the  $a_3 = 0$  gauge

$$\begin{aligned} \tilde{H} \equiv H(a_3 = 0) &= \frac{4\pi v}{g} \int d\rho \tilde{\mathcal{E}}(\rho), \\ \tilde{\mathcal{E}} &= \left[ (a'_1)^2 + (a'_2)^2 + \frac{1}{2\rho^2} (a_1^2 + a_2^2 - 1)^2 \right] \\ &+ 2 \left[ \rho^2 (h')^2 + 2(a_1^2 + a_2^2)h^2 + \frac{\lambda}{4g^2} \rho^2 (h^2 - 1)^2 \right] + \left( \frac{\zeta}{v} \right)^2 \left[ \rho^2 [(h'_1)^2 + (h'_2)^2] + 2(a_1 h_1 - a_2 h_2)^2 + \frac{\tau}{4g^2} \left( \frac{\zeta}{v} \right)^2 \rho^2 (h_1^2 + h_2^2 - 1)^2 \right] \\ &+ \kappa_1 [\rho^2 (f')^2 + 2Y^2] + \frac{4\kappa_2}{\rho^2} Y^2 [2\rho^2 (f')^2 + Y^2] \end{aligned} \quad (20)$$

with  $Y$  defined by (13). Equation (20) is the energy density functional that will be used in the subsequent study.

### III. STATIC SOLUTIONS

#### A. Overall picture

The energy (15), referenced from the monopole mass, is shown in Fig. 1 as a function of the baryon number. This plot

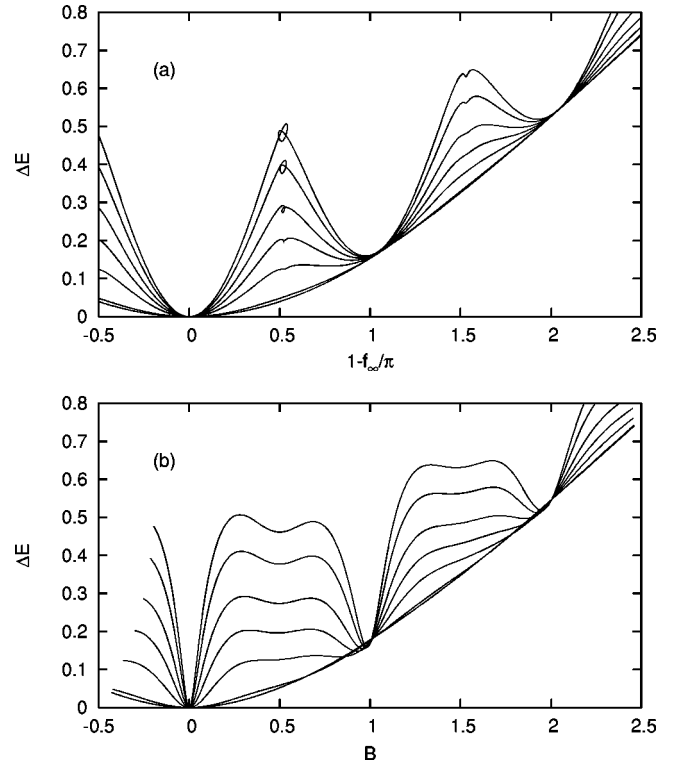


FIG. 2. If all mass scales are of the same order, the model becomes strongly coupled, which is demonstrated by the bifurcations in the energy dependence on  $f_\infty$ , plot (a). However bifurcations do not appear in energy versus the full baryon number  $B$ , plot (b). Curves plotted are for  $\kappa_1 = 1, \kappa_2 = 0.1, (\zeta/v)^2 =$  (top to bottom): 100, 70, 40, 22.36 (bifurcation point), 10, 1, 0.1.

is obtained numerically in a way outlined at the end of the previous section, and has always the same shape in the physical range of parameters  $F_\pi \ll \zeta \ll v$ . One observes that there indeed exist local minima at integer  $B$  separated by barriers centered at half-integer baryon number. The corresponding values of the energy are much lower than the mass of the bare Skymion, which in units of Fig. 1 is equal to 1.

It is interesting to note that the increase in  $\zeta/v$  gives rise to a considerable contribution to the Chern-Simons integral in Eq. (18), so that the curve  $B(f_\infty)$  is no longer monotonic. This results in the bifurcations exhibited on the  $E(f_\infty)$  plot of Fig. 2(a). However, even in the regime  $\zeta \gg v$ , the energy remains a single valued function of the baryon charge  $B$  [see Fig. 2(b)].

Let us come back to the physical range of parameters. As with the usual sphaleron barriers separating topologically inequivalent vacua in electroweak theory [9,10], the barriers separating states with integer baryon number are of topological origin. The vacuum configuration of the field  $\Psi$  is  $h_1 = 0, h_2 = 1$ , but at noninteger  $B$  the boundary conditions for the field  $\Psi$  lead to its departure from this vacuum. As mentioned above, the gauge invariant difference  $f_\infty - \beta$  is fixed and is equal to  $\pi$  along physical evolution paths. In the  $a_3 = 0$  gauge, integer  $B$  corresponds to  $f_\infty$  being a multiple of  $\pi$  [this is the case even for large  $\zeta/v$ , when the Chern-Simons term in Eq. (18) is generally not negligible]. Therefore, at



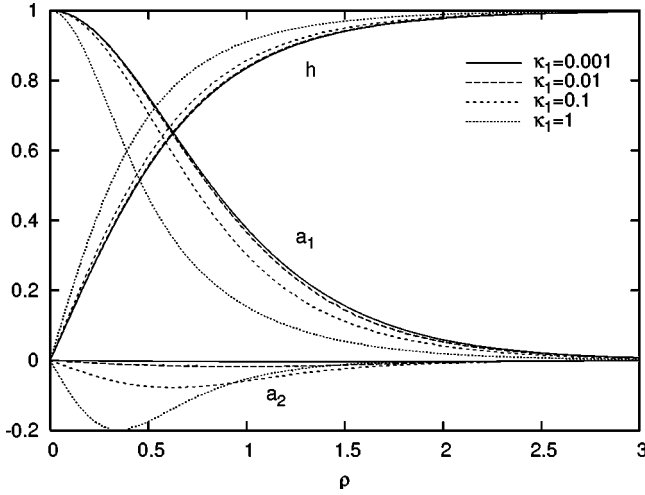


FIG. 3. At  $\zeta/v, \kappa_1/\kappa_2 \ll 1$  the profile functions  $a_1$ ,  $a_2$ , and  $h$  reach the free monopole limit and are not affected by the low-energy fields  $U$  and  $\Psi$ . We call this the spectator monopole regime. For all curves  $\kappa_2 = 0.1$ ,  $f_\infty = 0$ , and  $(\zeta/v)^2 = 10^{-7}$ . Normalization errors of Fig. 5, Ref. [7] are corrected.

integer  $B$  the boundary conditions for the  $h_1$  component of the  $\Psi$  field

$$h_1(0) = h_1(\infty) = 0$$

are compatible with the vacuum configuration, and the contribution of  $\Psi$  to the total energy is small.

This is no longer the case at noninteger  $B$ , when  $h_1(\infty) \neq 0$ , and  $h_1(0)$  is still 0. In other words  $h_1(\infty) \neq 0$  leads to a hedgehog configuration of the  $\Psi$  field at infinity, giving rise to a topological defect which in the spherically symmetric case is located at the origin. The contribution of this object to the energy and the  $E(B)$  curve giving the shape of the barrier can be evaluated in the spectator approximation, defined below.

## B. Spectator approximation

### 1. Energy of the Skyrme field

For the physical range of the parameters in the Hamiltonian (20), there is a hierarchy of “coupling constants” which reflects the presence of three physical scales in the problem

$$\frac{F_\pi^2}{8v^2} \equiv \kappa_1 \ll \frac{\zeta^2}{v^2} \ll \kappa_2 \sim 1,$$

which means that the monopole is so heavy that the effect of the other fields on it is negligible. In Ref. [7] this was called the spectator regime. It sets in at  $\kappa_1/\kappa_2 \lesssim 0.01$ ; see Fig. 3.

An important property of the spectator regime is that in the  $a_3 = 0$  gauge, the low-energy fields  $f$  and  $\Psi$  can be evaluated in the background field of the monopole, so they decouple and depend only on their own “coupling constants”  $\kappa_1/\kappa_2 \propto (F_\pi/v)^2$  and  $(\zeta/v)^2$ , respectively. As shown in Ref. [7], for  $\rho \gtrsim \rho_c \sim 1$  the solution  $f(\rho)$  has the simple form

$$f(\rho) = f_\infty + (\pi - f_\infty) \frac{\rho_c}{\rho}, \quad (21)$$

where

$$\rho_c \sim \frac{1}{4} \left| \ln \left( \frac{\kappa_1}{\kappa_2} \right) \right|. \quad (22)$$

Substituting Eqs. (21),(22) into the Hamiltonian (20), one obtains an estimate for the contribution of the Skyrme field to the energy in the presence of the monopole

$$E_{\text{Sk}} \sim \frac{4\pi v}{g} \kappa_1 \rho_c (\pi - f_\infty)^2 = \frac{\pi^3 F_\pi^2}{8vg} \left| \ln \frac{\kappa_1}{\kappa_2} \right| B^2, \quad (23)$$

where we made use of the relation

$$B = 1 - \frac{f_\infty}{\pi} = -\frac{\beta}{\pi} \quad (24)$$

valid in the spectator approximation.

### 2. Energy of extra Higgs field and the sphaleron

A similar estimate can be made for the contribution of the  $\Psi$  field. Supposing that  $h_2 = \text{const} = \cos \beta$  over all the volume, one obtains the following general equation for  $h_1$ :

$$(\rho^2 h_1')' - 2a_1^2 h_1 = \frac{\tau}{2g^2} \left( \frac{\zeta}{v} \right)^2 \rho^2 (h_1^2 - \sin^2 \beta) h_1. \quad (25)$$

Outside the monopole core, where  $a_1 = 0$ , one linearizes near  $h_1 = \sin \beta$ , and finds

$$h_1(\rho) - \sin \beta = \frac{\text{const}}{\rho} \exp \left( \pm \frac{\sqrt{\tau} \zeta}{gv} \sin \beta \cdot \rho \right).$$

The minus sign gives a solution that remains finite at  $\rho \rightarrow \infty$ . Taking into account that  $\zeta/v \ll 1$ , one obtains

$$h_1 = \sin \beta + \frac{\text{const}}{\rho}. \quad (26)$$

Now let us find  $h_1$  near the monopole center. For small  $\rho$ , Eq. (25) reduces to

$$(\rho^2 h_1')' = 2a_1^2 h_1.$$

Given that  $a_1(0) = 1$ , the solution near the center is

$$h_1 = \text{const} \times \rho. \quad (27)$$

To obtain the approximate solution everywhere, we match the two expressions for  $h_1$ , namely, Eqs. (26) and (27), as well as their derivatives, at  $\rho = \rho_0 \sim 1$  and find

$$h_1 = \frac{\rho}{2\rho_0} \sin \beta, \quad \rho < \rho_0, \\ h_1 = \left( 1 - \frac{\rho_0}{2\rho} \right) \sin \beta, \quad \rho > \rho_0. \quad (28)$$

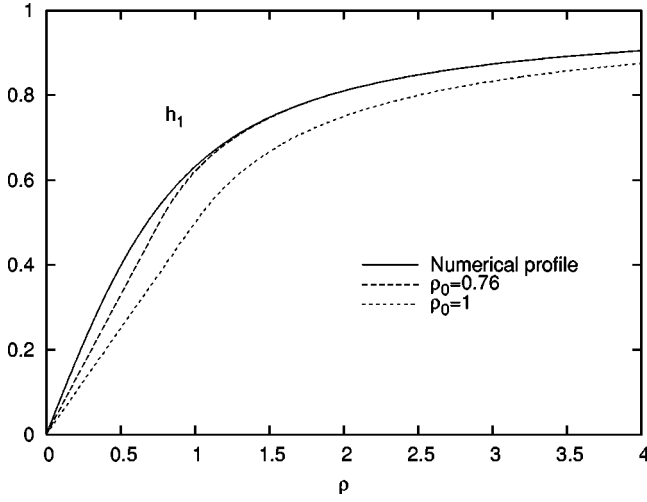


FIG. 4. The profile of the “sphaleron”—a topologically non-trivial configuration of the  $\Psi$  field as compared to the analytic estimate (28) at different values of  $\rho_0$ . The profile function  $h_2$  stays below  $2 \times 10^{-4}$ .

The best agreement with numerics (see Fig. 4) is obtained at  $\rho_0 \sim 0.76$ . Substituting Eq. (28) into the Hamiltonian (20), one finds for the contribution of the  $\Psi$  field to the energy

$$E_\Psi = E_{\text{sph}} \sin^2 \beta, \quad (29)$$

where

$$E_{\text{sph}} \sim 4\pi \frac{\zeta^2}{gv} \frac{\rho_0}{2} \quad (30)$$

which is about 50% higher than the actual numerical value; see Fig. 5. It is worth noting that, similarly to the Skymion,

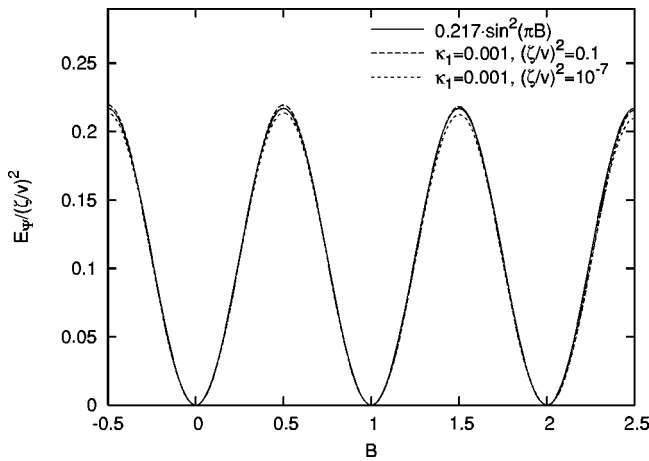


FIG. 5. After normalization by prefactor  $(\zeta/v)^2$ , the contribution of the  $\Psi$  field to the full energy (20) is in perfect agreement with the  $\sin^2 \pi B$  curve, Eq. (31). However, the normalized sphaleron mass (30) is estimated to be  $\rho_0/2 \sim 0.38$  instead of the actual value 0.217. The curves shown represent maximal deviations from Eq. (31) over all numerical data available. In the physical limit, this new periodic structure dominates the total energy.

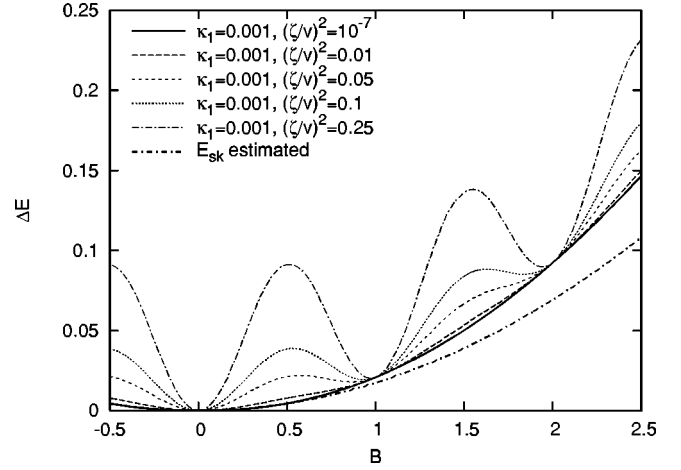


FIG. 6. In the spectator approximation the energy  $\Delta E$ , Eq. (32), is equal to “thin” Skymion energy  $E_{\text{Sk}}$ , solid line (identical to Fig. 1 of Ref. [7]), plus the periodic contribution of the  $\Psi$  field, Fig. 5. Once the latter is of order of  $\zeta^2/v$  while  $E_{\text{Sk}} \sim F_\pi^2/v$ , see Eqs. (23) and (30), in the physical limit the barriers are much higher than the Skymion energy, which prevents Skymion decay along the static path. The lower line is the estimate of  $E_{\text{Sk}}$ , Eq. (23).

the solution is stabilized against shrinking to zero size only by the monopole core. For a pointlike monopole  $\rho_0 \rightarrow 0$  and  $E_{\text{sph}} \rightarrow 0$ .

Taking into account the relation (24), valid in the spectator approximation, one can rewrite Eq. (29) as

$$E_\Psi(B) = E_{\text{sph}} \sin^2 \pi B \quad (31)$$

which is indeed similar [10] to the periodic vacuum structure of the standard electroweak theory. Local maxima of  $E_\Psi(B)$  at half-integer  $B$  correspond to sphaleronlike solutions in the presence of the monopole.

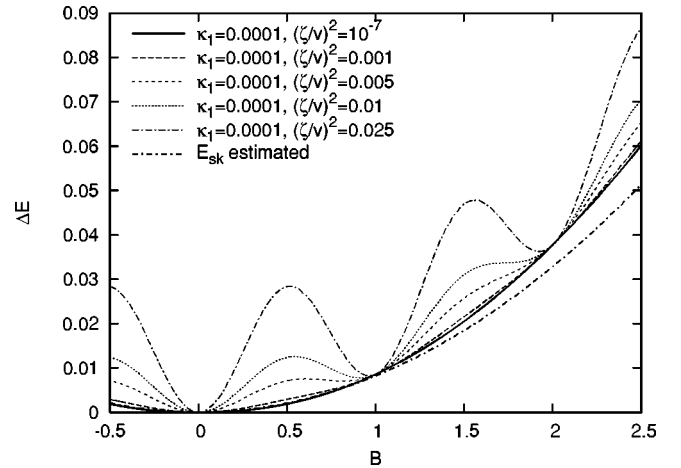


FIG. 7. Same as Fig. 6 for smaller  $\kappa_1 = 8F_\pi^2/v^2$  and identical ratios of  $\zeta^2/F_\pi^2$ . Note that the relative height of the barriers remains similar to Fig. 6 and the estimate for  $E_{\text{Sk}}$  gets closer to the numerical curve (solid line), which demonstrates the onset of the spectator limit.

### 3. Total energy

In the spectator approximation, the curve  $\Delta E(B) = E(B) - M_{\text{mon}}$  is the sum of the two contributions (23) and (31)

$$\Delta E(B) = C_1 \frac{F_\pi^2}{g\nu} B^2 + C_2 \frac{\xi^2}{g\nu} \sin^2 \pi B, \quad (32)$$

where approximate expressions for  $C_1$  and  $C_2$  are determined by Eqs. (23) and (30), respectively, and their values are of order one (up to logarithm). Comparison with numerical data is shown in Figs. 6,7. Even though both terms are suppressed by the “GUT” VEV  $\nu$ , the sphaleron remains much heavier than the Skyrmin bound state, since  $\xi \gg F_\pi$ . In other words, the energy gain from the decay of the Skyrmin bound state is of the order of  $E_{\text{sk}} \sim F_\pi^2/(g\nu)$ , which is much less than the barrier height  $E_{\text{sph}} \sim \xi^2/(g\nu)$ . Therefore, the introduction of intermediate energy scale prohibits the decay of the Skyrmin-like solution along the minimal energy path.

## IV. CONCLUSIONS

In the model employed in Ref. [7] the axial vector mass was unphysically light. Here we have rectified this by the introduction of an extra Higgs field with VEV  $\xi$  which is meant to mimic the electroweak energy scale. A continuous set of static solutions with varying baryon number is found numerically. The extra Higgs field gives rise to energy barriers along this set.

In the physical range of the parameters of the model we have a complete understanding of its static properties, with good agreement between analytic estimates and numerical data. The monopole is so heavy that its deformations due to interaction with the rest of the system are negligible. In a convenient gauge  $a_3 = 0$ , the Skyrme field and extra Higgs

field decouple,<sup>2</sup> their shapes are defined by interaction with the background monopole. Both the Skyrmin and sphaleron are strongly deformed by the monopole: their asymptotic behavior emerges immediately outside the monopole core, and they collapse to the origin in the limit of pointlike monopole. The energies of these solutions are suppressed by the largest VEV.

The existence of the barrier between local minima of static energy does not mean at all that the Skyrmin decay cannot take place in the Skyrmin-monopole scattering in the model at hand. Here the incoming “bare” Skyrmin becomes unstable in the presence of the monopole and thus does not correspond to any static solution. The “bare” Skyrmin is much heavier than its “thin” static counterpart with unit baryon charge, the latter concentrating in the vicinity of the monopole core. The actual decay of the Skyrmin is a dynamical process, not a quasistationary one, and the corresponding path goes much higher than the  $E(B)$  curve of Fig. 1. In other words, to get into one of the stable local minima at integer  $B$ , the Skyrmin must release almost all of its energy while preserving the baryon number, which in the monopole background is no longer stabilized by boundary conditions at infinity. This appears to be highly unlikely.

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<sup>2</sup>Except that they are related through the boundary condition (19) at spatial infinity.

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